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MORE ON SIMON'S TWO ECHELON MODEL

U.S. ARMY
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TECHNICAL REPORT

BY

W. KARL KRUSE

NOVEMBER 1977

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19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This paper discusses some further analysis by the IRO of a two echelon inventory model developed by R. M. Simon, and subsequently corrected by Kruse and Kaplan. The Kruse and Kaplan expressions, as published, were quite complex, and much additional manipulation was required to put them in computational form. In doing this, it was discovered that these were reducible to a simple and revealing form. Moreover, it was also learned that the Simon

expressions reduced to the same simple form despite the logical flaw in their development. Reasons for the identity are presented in this paper. Appendices A and B contain the algebraic reductions of Simon's and Kruse and Kaplan's expressions.

Kruse and Kaplan [1] published a note "Comments on Simon's Two Echelon Model" which identifies a logical flaw in a paper by R. M. Simon [2] which modelled a two echelon inventory system composed of a number of independent bases using S-1,S replenishment policies, and a common depot using a more general R,Q type policy. The failure process at each base was assumed Poisson with rate λ_j. A given failure was repaired at the base j with probability r_j, or sent to the depot with probability $1-r_j$, whereupon it was repaired with probability ρ, or condemned with probability $1-\rho$. All repair actions were taken to be started immediately without any batching. Repair and supply lead times were all deterministic.

The essence of the model was the expression for the probability distribution of dues-out to base j (amount on backorder at the depot for base j) at time t. Naturally, this is in part dependent upon the demand in the depot's supply lead time, i.e., the time it takes the depot to receive replenishment stocks from its source of supply. Simon contended that part of the demands in the depots supply lead time could be ignored because they had no effect at all on dues out at time t. Specifically, those demands in the lead time which are associated with carcasses that can be repaired before t could be ignored since each of those demands "furnishes a serviceable carcass by time t in place of the carcass it takes, and does not alter the sequence of relevant demands." Kruse and Kaplan pointed out that although total dues out at t were unaffected by these demands, they, nonetheless, did affect to whom those dues-out were due. A simple example supported this claim, and a different analysis was done using many of Simon's methods. However, while Kruse and Kaplan were correct in claiming that actual dues-out were dependent on these demands, they were nevertheless incorrect in believing that this necessarily meant that the probability distribution did too. In fact, for the particular set of assumptions and parameters in Simon's paper, both Simon's and Kruse and Kaplan's results are identical. This may be shown formally by algebraically reducing both results.

The reduced result is quite compact and revealing. It says simply that if total dues-out at the depot at time t are, say b, then the amount due out to base j is binomially distributed with parameters b, and $\lambda_j(1-r_j)/\sum_k \lambda_k(1-r_k)$, the probability that a given demand is from base j.

The key to this simple result is that for the assumptions made it follows that the probability that a carcass sent to the depot is reparable does not depend upon the base from which the carcass came. Thus, the likelihood of a given demand on the depot coming from base j does not depend on whether that demand is associated with a reparable carcass. This means that the probability a given due out is for base j is the same as the probability a given demand on the depot is from base j.

If the assumptions are relaxed so that ρ is base dependent, say ρ_j, then the Kruse and Kaplan model may be easily modified. Particularly in case 2B set

$$\Pr[D_j(t_a,t_b) = d_j(t_a,t_b) \mid D_o^C(t_a,t_b) = d_o^C(t_a,t_b),\ D_o^D(t_a t_b) = d_o^D(t_a,t_b)]$$

$$= \sum_{k=o}^{d_j(t_a,t_b)} \binom{d_o^C(t_a,t_b)}{k} P_C^k (1-P_C)^{d_o^C(t_a,t_b)-k} \binom{d_o^D(t_a,t_b)}{d_j(t_a,t_b)-k}$$

$$\cdot\ P_D^{d_j(t_a,t_b)-k} (1-P_D)^{d_o^D(t_a,t_b)-d_j(t_a,t_b)+k}$$

where

$$P_C = \frac{(1-r_j)(1-\rho_j)\lambda_j}{\sum_k (1-r_k)(1-\rho_k)\lambda_k},$$

$$P_D = \frac{(1-r_j)\rho_j\ \lambda_j}{\sum_k (1-r_k)\rho_k\lambda_k},$$

and

$$\lambda_o^D = \sum_j (1-r_j)\rho_j\lambda_j .$$

With $P_C = P_D$, i.e. if $\rho_j = \rho$ for all j, then this probability reduces to the form of the Kruse and Kaplan model.

Appendices A and B show the algebraic reductions of Simon's and Kruse and Kaplan's expressions.

REFERENCES

1 W. Karl Kruse and Alan J. Kaplan, "Comments on Simon's Two Echelon Model," Operations Research 21, No. 6.

2 Richard M. Simon, "Stationary Properties of a Two Echelon Inventory Model for Low Demand Items," Operations Research 19, No. 3.

APPENDIX A

REDUCTION OF SIMON'S EQUATIONS (12) AND (13)

Equation (13), the probability that the number due out to base j equals o, is a special case since it must include those situations where the depot has stock on hand as well as those situations when the depot has dues-out but none are for base j. This second set is the most interesting analytically, and in fact is the same form as equation (12). Consequently, we will limit this demonstration to the reduction of equation (12). The reduced form of (13) follows obviously.

We have from Simon (12) that

(A1) $\Pr[\text{Due out to base } j = d \mid X(t_a) = s_o + k] = \Pr[E_j(t) = d \mid X(t_a) = s_{o+k}]$

$$= \sum_{d_o = s_o+k+d}^{\infty} \Pr[D_o^C(t_a,t) + D_o^D(t_b,t) = d_o]$$

$$\sum_{d_j=d}^{s_o+k+d} \left\{ \binom{d_j}{d} \binom{d-d_j}{s_o+k-d_j+d} \Big/ \binom{d_o}{s_o+k} \right\} \binom{d_o}{d_j} P_j^{d_j} (1-P_j)^{d_o-d_j}$$

where $P_j = \dfrac{(1-r_j)\lambda_j}{\sum_k (1-r_k)\lambda_k}$.

Since the inner summation is equal to

$$\sum_{d_j=d}^{s_o+k+d} \binom{s_o+k}{d_j-d} \binom{d_o-s_o-k}{d} P_j^{d_j} (1-P_j)^{d_o-d_j}$$

$$= \sum_{d_j=o}^{s_o+k} \binom{s_o+k}{d_j} \binom{d_o-s_o-k}{d} P_j^{d_j+d} (1-P_j)^{d_o-d_j-d}$$

$$= \binom{d_o-s_o-k}{d} P_j^d (1-P_j)^{d_o-s_o-k-d} ,$$

we get

(A2) $$\Pr[E_j(t) = d \mid X(t_a) = s_o+k] =$$

$$\sum_{d_o=s_o+k+d}^{\infty} \Pr[D_o^C(t_a, t + D_o^D(t_b, t) = d_o] \binom{d_o-s_o-k}{d} P_j^d \, (1-P_j)^{d_o-s_o-k-d}$$

$$= \sum_{d_o=d}^{\infty} \Pr[D_o^C(t_a, t) + D_o^D(t_b, t) = d_o + s_o + k] \binom{d_o}{d} \, P_j^d \, (1-P_j)^{d_o-d}$$

$$= \sum_{b=d}^{\infty} \Pr[\text{Depot backorders at } t = b] \binom{b}{d} \, P_j^d \, (1-P_j)^{b-d} \quad .$$

APPENDIX B

REDUCTION OF KRUSE AND KAPLAN'S EQUATIONS (3) AND (4)

First consider Kruse and Kaplan's Case 2A where $D_o^C(t_a,t_b) < X(t_a)$

$$\Pr[\text{Due out to base } j = d \mid X(t_a) = x,\ D_o^C(t_a,t_b) = d_o^C,\ D_o(t_b,t) = d_o]$$

$$= \Pr[E_j(t) = d \mid X(t_a) = x,\ D_o^C(t_a,t_b) = d_o^C,\ D_o(t_b,t) = d_o]$$

$$= \sum_{d_j=d}^{x+d-d_o^C} \left\{ \binom{d_o-x+d_o^C}{d} \binom{x-d_o^C}{d_o-d} \bigg/ \binom{d_o}{d_j} \right\} \binom{d_o}{d_j} P_j^{d_j} (1-P_j)^{d_o-d_j} ,$$

where the elements of the combinatorials of their equation (3) have been rearranged and P_j is as defined in Appendix A. Then

$$\Pr[E_j(t) = d \mid x(t_a) = x,\ D_o^C(t_a,t_b) = d_o^C,\ D_o(t_b,t) = d_o]$$

$$= \sum_{d_j=d}^{x+d-d_o^C} \binom{d_o-x+d_o^C}{d} \binom{x-d_o^C}{d_j-d} P_j^{d_j} (1-P_j)^{d_o-d_j}$$

$$= \binom{d_o-x+d_o^C}{d} \sum_{d_j=o}^{x-d_o^C} \binom{x-d_o^C}{d_j} P_j^{d_j+d} (1-P_j)^{d_o-d_j-d}$$

$$= \binom{d_o+d_o^C-x}{d} P_j^d (1-P_j)^{d_o+d_o^C-x-d} . \qquad \text{(B1)}$$

Now consider Kruse and Kaplan's case 2B where $D_o^C(t_a,t_b) \geq X(t_a)$. With their definition of $U_j^2(t)$ we have that

$$E_j(t) = U_j^2(t) + D_j(t_b,t)$$

Now

$$\Pr[U_j^2(t) = d \mid X(t_a) = x,\ D_o^C(t_a,t_b) = d_o^C,\ D_o^D(t_a,t_b) = d_o^D]$$

$$= \sum_{d_j=d}^{d_o^D+x+d} \left\{ \binom{d_j}{d_j-d} \binom{d_o^D+d_o^C-d_j}{x+d_o^D-d_j+d} \Big/ \binom{d_o^D+d_o^C}{x+d_o^D} \right\} \binom{d_o^C+d_o^D}{d_j} P_j^{d_j} (1-P_j)^{d_o^C+d_o^D-d_j}$$

$$= \sum_{d_j=d}^{x+d_o^D+d} \binom{x+d_o^D}{d_j-d} \binom{d_o^C-x}{d} P_j^{d_j} (1-P_j)^{d_o^C+d_o^D-d_j}$$

$$= \sum_{d_j=o}^{x+d_o^D} \binom{x+d_o^D}{d_j} \binom{d_o^C-x}{d} P_j^{d_j+d} (1-P_j)^{d_o^C+d_o^D-d_j-d}$$

$$= \binom{d_o^C-x}{d} P_j^d (1-P_j)^{d_o^C-x-d}$$

Note this does not depend on D_o^D

Now convoluting $U_j^2(t)$ and $D_j(t_b,t)$ to get $E_j(t)$ we have

$$\Pr[E_j(t) = d \mid X(t_a)=x,\ D_o^C(t_a t_b) = d_o^C,\ D_o(t_b,t) = d_o]$$

$$= \sum_{k=o}^{d} \Pr[U_j^2(t)=k \mid X(t_a)=x,\ D_o^C(t_a,t_b) = d_o^C]$$

$$\Pr[D_j(t_b,t) = d - k \mid D_o(t_b,t) = d_o]$$

$$= \sum_{k=o}^{d} \binom{d_o^C - x}{k} P_j^k (1-P_j)^{d_o^C-x-k} \binom{d_o}{d-k} P_j^{d-k} (1-P_j)^{d_o-d+k}$$

$$= \binom{d_o+d_o^C-x}{d} P_j^d (1-P_j)^{d_o+d_o^C-x-d} \qquad \text{(B2)}$$

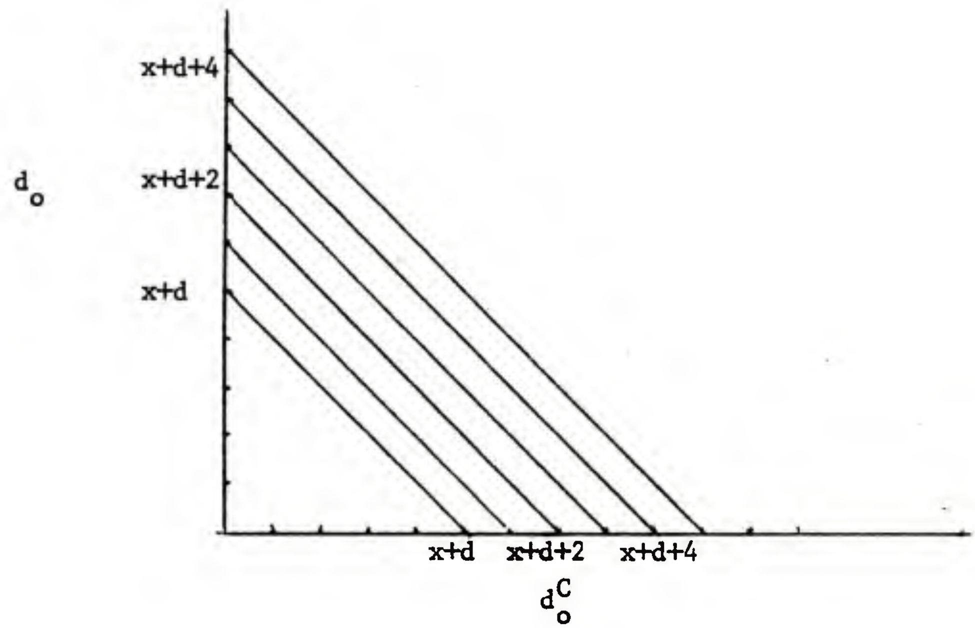
Note that this is the exact form of (B1)

Combining (B1) and (B2) we have

$$\Pr[E_j(t) = d | X(t_a) = x]$$

$$= \sum_{d_o^C=o}^{\infty} \sum_{d_o=\max(o,x-d_o^C+d)}^{\infty} \Pr[D_o^C(t_a,t_b)=d_o^C]\Pr[D_o(t_b,t) = d_o]\binom{d_o+d_o^C-x}{d}P_j^d(1-P_j)^{d_o+d_o^C-x-d}$$

Consider the region of summation



All of lines $d_o + d_o^C = x + b$ for $b \geq d$ cover the region of summation. Consequently we may sum along each of these lines .

Doing that we have

$$\Pr[E_j(t) = d \mid X(t_a) = x]$$

$$= \sum_{b=d}^{\infty} \Pr[D_o^C + D_o = x + b] \binom{b}{d} P_j^d (1-P_j)^{b-d}$$

$$= \sum_{b=d}^{\infty} \Pr[\text{Depot backorder at } t=b] \binom{b}{d} P_j^d (1-P_j)^{b-d} \qquad \text{(B3)}$$

for $d > o$

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